Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International GCSE

Time 2 hours

Paper reference **4PM1/02**

Further Pure Mathematics
PAPER 2

Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

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P 7 1 6 6 6 A 0 1 3 6

Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times$ slant height

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$ $\quad$ $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$ $\quad$ $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \qquad \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 Find the set of values of k for which the equation

$$2kx^2 + 5kx + 5k - 3 = 0 \qquad \text{where } k \neq 0$$

has real roots.

(4)

(Total for Question 1 is 4 marks)

2 A particle P moves along the x-axis. At time t seconds, the displacement, x metres, of P from the origin O is given by

$$x = t^4 - 13.5t + 12$$

(a) Find the velocity, in m/s, of P when $t = 3$

(2)

(b) Find the value of t for which P is instantaneously at rest.

(2)

(c) Find the acceleration, in m/s^2, of P when $t = 2$

(2)

Question 2 continued

(Total for Question 2 is 6 marks)

3 O, A and B are fixed points such that

$$\overrightarrow{OA} = (p\mathbf{i} - 4\mathbf{j}) \qquad \overrightarrow{OB} = \mathbf{i} + (2p + 1)\mathbf{j}$$

Given that $\sqrt{2}\left|\overrightarrow{OA}\right| = \left|\overrightarrow{OB}\right|$ and $p > 0$

(a) find the value of p

(4)

Using this value of p

(b) find a unit vector that is parallel to $\overrightarrow{AB}$

(5)

Question 3 continued

(Total for Question 3 is 9 marks)

4

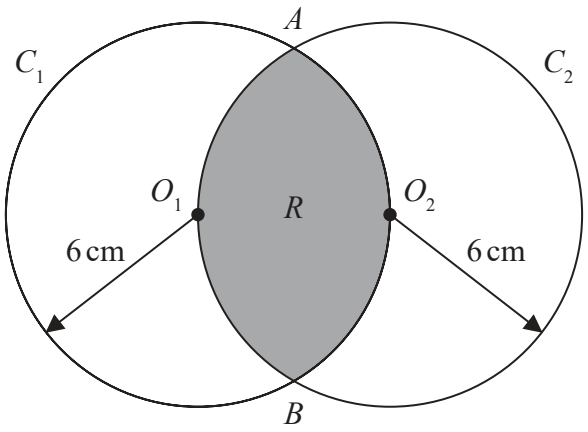

Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows two circles, C_1 and C_2, each with a radius of 6 cm.

The centre of C_1 is O_1 such that O_1 lies on C_2

The centre of C_2 is O_2 such that O_2 lies on C_1

The circles intersect at the points A and B and enclose the region R, shown shaded in Figure 1

The area of region R is P cm^2

Find the exact value of P, giving your answer in the form $a\pi - b\sqrt{c}$ where a, b and c are integers.

(7)

Question 4 continued

Question 4 continued

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Question 4 continued

(Total for Question 4 is 7 marks)

5 The roots of the quadratic equation $2x^2 + (6 + 2p)x + 2p = 0$ are α and β

(a) Write down an expression in terms of p for

(i) $\alpha + \beta$ (ii) $\alpha\beta$

(2)

(b) Show that $(\alpha - \beta)^2 = 9 + 2p + p^2$

(4)

Given that $(\alpha - \beta) = 3$

(c) find the possible values of p

(3)

Question 5 continued

Question 5 continued

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Question 5 continued

(Total for Question 5 is 9 marks)

6 (a) Using a formula from page 2, show that $\cos 2A = 1 - 2\sin^2 A$

(2)

The finite region R is bounded by the curve with equation $y = 3 + 2\sin x$, the x-axis, the y-axis and the line with equation $x = \frac{\pi}{4}$

The region R is rotated through 360° about the x-axis.

(b) Use calculus to find the volume of the solid generated.
Give your answer to the nearest integer.

(6)

Question 6 continued

(Total for Question 6 is 8 marks)

7 (i) (a) Using a formula from page 2, show that

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

(2)

Given that $\tan 2\alpha = 1$

(b) show that $\tan\alpha = a \pm \sqrt{b}$ where a and b are integers whose values need to be found.

(3)

(ii) (a) Using formulae from page 2, show that $\cos(x - 30)^\circ = \sin(x + 30)^\circ$ can be written as $\tan x^\circ = 1$

(4)

(b) Hence, or otherwise, solve

$$\cos(2y - 30)^\circ = \sin(2y + 30)^\circ \qquad \text{for } -90 < y \leqslant 90$$

(2)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 11 marks)

8

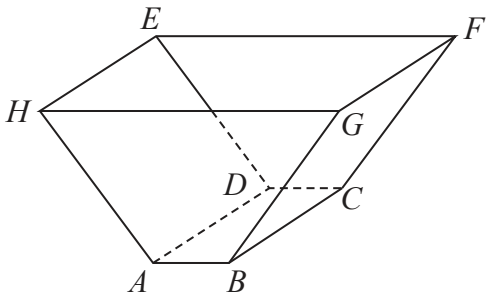

Diagram **NOT** accurately drawn

Figure 2

Figure 2 shows a waste paper basket in the shape of a right prism with 5 faces and a cross section that is a trapezium. The top, *EFGH*, of the waste paper basket is open.

The base of the prism *ABCD* is a rectangle with

$$AB = DC = 2x\,\text{cm} \text{ and } AD = BC = h\,\text{cm}$$

The cross sections *HGBA* and *EFCD* are such that

$$EF = HG = 8x\,\text{cm} \text{ and } AH = BG = CF = DE = 5x\,\text{cm}$$

The top, *EFGH*, of the waste paper basket is such that

$$EH = FG = h\,\text{cm}$$

The volume of the waste paper basket is $2250\,\text{cm}^3$

The total surface area of the 5 faces of the waste paper basket is $S\,\text{cm}^2$

(a) Show that $S = 40x^2 + \dfrac{1350}{x}$

(5)

Given that x can vary,

(b) use calculus, to find, to 3 significant figures, the value of x for which S is a minimum.
Justify that this value of x gives a minimum value of S

(5)

(c) Find, to 3 significant figures, the minimum value of S

(2)

Question 8 continued

Question 8 continued

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Question 8 continued

(Total for Question 8 is 12 marks)

9 The straight line L_1 passes through the point A with coordinates (4, 7) and has gradient m, where $m < 0$

Another straight line L_2 is perpendicular to L_1 and passes through the point B with coordinates (4, k) where $k \neq 7$

The lines L_1 and L_2 intersect at the point C.

Given that the y coordinate of C is Y

(a) show that $Y = \dfrac{7 + m^2k}{m^2 + 1}$

(7)

Given that the triangle ABC is isosceles,

(b) find the value of m

(5)

Question 9 continued

Question 9 continued

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Question 9 continued

(Total for Question 9 is 12 marks)

10 Solve the equation

$$\log_4 x + \log_{16} x + \log_2 x = 10.5$$

Show your working clearly.

(5)

Question 10 continued

(Total for Question 10 is 5 marks)

11 A curve C has equation

$$y = \frac{(2a-1)x+1}{ax-6} \qquad \text{where } a \text{ is a constant and } x \neq \frac{6}{a}$$

(a) Find $\frac{dy}{dx}$

(3)

The curve crosses the y-axis at the point A.

The normal to C at the point A is the line l with equation $66y - 72x + 11 = 0$

Show that

(b) (i) $a = 3$

(4)

(ii) the equation of C is $y = \frac{5x+1}{3x-6}$ where $x \neq 2$

(1)

(c) Using the axes on the opposite page, sketch C, showing clearly the asymptotes with their equations and the coordinates of the points where C crosses the coordinate axes.

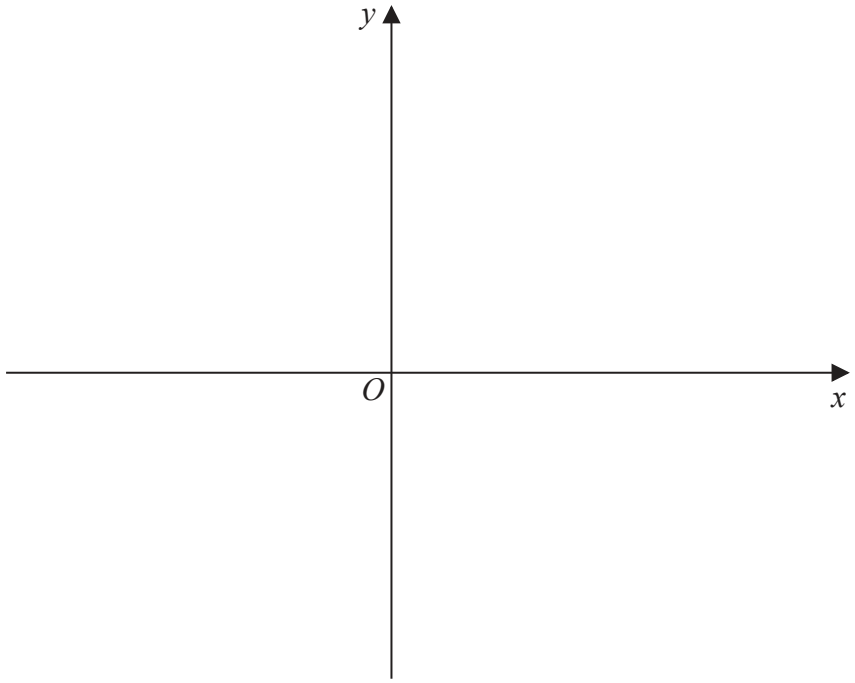
(5)

The line l meets C again at the point D.

(d) Find the x coordinate of D.
Give your answer as an improper fraction.

(4)

Question 11 continued



Question 11 continued

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Question 11 continued

Question 11 continued

(Total for Question 11 is 17 marks)

TOTAL FOR PAPER IS 100 MARKS

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